Mean-Field limit of the Bose-Hubbard model in high dimension

Study: large system of quantum bosons

Usually: many-body $N \to \infty$ mean field:

$$H_N \coloneqq \sum_{i=1}^{N} (-\Delta_i) + \frac{1}{N} \sum_{1 \leqslant i < j \leqslant N} w(X_i - X_j) \quad \text{acting on } L^2(\mathbb{R}^d, \mathbb{C})^{\otimes_+ N}$$

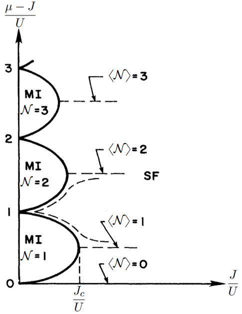
Statistical description of the interaction: $h^{\varphi}_{\text{Hartree}} = -\Delta + |\varphi|^2 \star w$ with $\varphi \in L^2(\mathbb{R}^d)$

Bose-Hubbard model: interacting bosons on a lattice

- phase transition: Mott-insulator \ Superfluid
- Mean field justified when $d \to \infty$, effective in $d = 3$

Results: mean field limit as $d \to \infty$

- Dynamics and ground state energy
- Strong and local particle interactions



$$H_{\text{Bose-Hubbard}} \coloneqq -\frac{J}{2d} \sum_{\substack{x,y \in \Lambda \\ x \sim y}} a_x^\dagger a_y + (J - \mu) \sum_{x \in \Lambda} \mathcal{N}_x + \frac{U}{2} \sum_{x \in \Lambda} \mathcal{N}_x (\mathcal{N}_x - 1)$$

$$h^{\varphi}_{\text{Mean-field}} \coloneqq -J\left(\overline{\alpha_\varphi} a + \alpha_\varphi a^\dagger - |\alpha_\varphi|^2\right) + (J - \mu)\mathcal{N} + \frac{U}{2}\mathcal{N}(\mathcal{N} - 1) \text{ with } \alpha_\varphi \coloneqq \langle \varphi | a \varphi \rangle$$